

Math 409: General Topology

Fall 2019

MW - 2:00 - 3:15 pm

Harris Hall 4145

Instructor: Brent Cody

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Prerequisite: A “C” or better in MATH 407 or equivalent

Office Hours: M 3:30 - 4:30 pm

T 10:00 - 11:00 am

Description:

Foundations and fundamental concepts of point-set topology. Topological spaces, continuity, convergence, connected sets, compactness, product spaces, quotient spaces, function spaces, separation properties.

About the Course:

In mathematics, what does the word “space” mean? Of course the answer depends on the context. Topology is essentially about generalizing the basic notions of geometry and calculus, such as **point**, **space**, **distance**, **convergence** and **continuity**. For example:

- Recall that if a sequence of reals $\langle x_n \rangle$ converges to a point $a \in \mathbb{R}$ then the sequence cannot converge to any other distinct point. In other words, if $\lim_{n \rightarrow \infty} x_n = a$ and $b \neq a$ then $\lim_{n \rightarrow \infty} x_n \neq b$. We say that the reals \mathbb{R} under the usual notion of convergence satisfies the *unique limit property*.
- What properties of the reals \mathbb{R} allow us to prove that \mathbb{R} has the unique limit property?
- Which “spaces” also have the unique limit property?
- Is there some alternative notion of convergence for sequences of reals such that a sequence $\langle x_n \rangle$ can converge to two distinct points?

Topics:

- (1) Review of logic, basic set theory and functions
- (2) Countable and uncountable sets, infinite unions/intersections
- (3) Continuity and convergence in \mathbb{R} , \mathbb{R}^2 , ...
- (4) Metric spaces
- (5) Topological spaces
- (6) The Euclidean topology, basis for a topology
- (7) Limit points, closure and separation axioms
- (8) Homeomorphisms
- (9) Continuous maps
- (10) More metric spaces
- (11) Compactness - the Heine-Borel Theorem: a subset of \mathbb{R} is compact if and only if it is closed and bounded.
- (12) Products of spaces - Cantor space, Hilbert cube, Urysohn’s Theorem
- (13) Tychonoff’s Theorem
- (14) Quotient spaces
- (15) Fundamental group

Grading:

The assignments for the course are broken up into four categories.

(1) Assignments: Homework assignments will be posted on blackboard each week. You can work together on homework, but your solutions should be your own (do not directly copy). HW will be collected and graded. There will also be less frequent in-class assignments.

(2) Midterm Exam: There will be a midterm exam about halfway through the semester. The midterm exam will consist of a take-home portion and an in-class portion. You are allowed to work together on the take-home portion, but solutions should be your own.

(3) Final Exam: The final exam will be **cumulative** and will also consist of a take-home portion and an in-class portion. The final exam will take place on

Monday, December 16, 12:30 pm – 3:20 pm

in our regular classroom, Harris Hall 4145.

The usual 10-point grading scale will be used to determine course grades (90–100 is an A, 80–89 is a B, etc.). Here is a chart indicating the weights of assignments.

Assignments	36%
Midterm Exam	32%
Final Exam	32%
Total	100%

Grading Policies:

- I will drop two of your lowest “assignment” grades.
- **Make-up policy:** Under normal circumstances I do not make a distinction between excused absences and unexcused absences. I do not give make-up quizzes nor do I accept late homework, rather I drop your lowest quiz and homework grades (as outlined above). If you miss the final exam for a legitimate reason (i.e. a documented illness or emergency) then I can give you a grade of incomplete (I) for the course, and you will need to make up the missed exam.

Course Policies:

- Missing more than 5 classes may result in an automatic F.
- No cheating on quizzes, tests or exams will be tolerated. If you are caught cheating on an assignment you will receive a 0 on that assignment.

All cellphones and electronic devices must be put away during class, including during quizzes and exams. The use of a cellphone during quizzes or exams will be considered cheating unless you ask me first.

Books:

I recommend that you download all of the freely available books listed below.

TOPOLOGY TEXTBOOKS:

Required Textbook (freely available)

- *Topology Without Tears*, by Sidney A. Morris
<http://www.topologywithouttears.net/topbook.pdf>

Time permitting we might cover Section 1.1 of this book (freely available)

- *Algebraic Topology*, by Alan Hatcher
 - <https://pi.math.cornell.edu/~hatcher/AT/ATpage.html>
 - Contains material on the fundamental group of a space.

Optional books available online through the VCU library

- *Topology: General & Algebraic*, by D. Chatterjee
 - http://search.library.vcu.edu/VCU:all_scope:VCU_ALMA51409501790001101
 - This motivates the beginning material pretty well, but leaves a lot of details out on more advanced material.
 - You can download a 14 day free trial of this book through the VCU library, but you can read it online as much as you want.
- *A Course in Point Set Topology*, by John B. Conway
 - http://search.library.vcu.edu/VCU:all_scope:VCU_ALMA51447752270001101
 - This has a good appendix on sets, functions, real numbers and countable/uncountable sets.
- *Topology*, by Stefan Waldman
 - http://search.library.vcu.edu/VCU:all_scope:VCU_ALMA51450089810001101
 - This one is fairly advanced.
- *Topology*, by Marco Manetti
 - http://search.library.vcu.edu/VCU:all_scope:VCU_ALMA51462378960001101
 - This book is also advanced.

Not freely available

- *Topology*, 2nd Ed by James R. Munkres — This is the most widely used topology book.
- *Foundations of Topology*, by C. Wayne Patty
- *Essential Topology*, by Martin D. Crossley
 - Each topic that it covers is expertly motivated.

BASIC LOGIC, SET THEORY AND PROOF TECHNIQUES:

Book of Proof (Second Edition) by Richard Hammack

- A pdf of the entire textbook is available at:
<http://www.people.vcu.edu/~rhammack/BookOfProof/index.html>

- You can order a paperback copy for \$13.75 from Amazon.
<http://www.amazon.com/Book-Proof-Richard-Hammack/dp/0989472108>

Important Dates:

Friday, November 1 Last day to withdraw from a course with a mark of “W”

UNIVERSITY WIDE POLICIES

Students should visit <http://go.vcu.edu/syllabus> and review all syllabus statement information. The full university syllabus statement includes information on safety, registration, the VCU Honor Code, student conduct, withdrawal and more.